) Evaluate $\int_{0}^{3}\int_{-1}^{1}\int_{2}^{4}(y-xz)\,dz\,dy\,dx$.	6	L2	5	1
) Evaluate $\int_{0}^{\infty}\int_{0}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$ by changing to polar coordinates. Hence deduce that $\int_{0}^{\infty} e^{-x^2}\,dx=\frac{\sqrt{\pi}}{2}$.	7	L2	5	2
) Find the area between the parabolas $y^2=4ax$ and $x^2=4ay$ by using double integration.	7	L2	5	1

3loom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

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3. a) Test for consistency and hence solve the system of equations by the Gauss elimination method:
$4x+y+z=4$
$x+4y-2z=4$
$3x+2y-4z=6$ — 6 L2 1

b) State Comparision test. Test for convergence of the series $\frac{1.2}{3.4.5}+\frac{2.3}{4.5.6}+\frac{3.4}{5.6.7}+\ldots\ldots\ldots\infty$. — 7 L2 2

c) State Cauchy's root test. Test for convergence of the series:
$1+\frac{2}{3}x+\left(\frac{3}{4}\right)^2x^2+\left(\frac{4}{5}\right)^3x^3+\ldots\ldots\ldots\infty,\ (x>0)$ — 7 L2 2

Unit – II

4. a) Define Jacobian of a transformation. If $x+y+z=u, y+z=uv$ and $z=uvw$ then find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$. — 6 L2 4

b) With usual notation prove that $\tan\varphi=r\frac{d\theta}{dr}$. — 7 L1 3

c) State Lagrange's mean value theorem. Verify Lagrange's mean value theorem for the function $f(x)=\sin^{-1}x$ in $[0,1]$. — 7 L1 3

5. a) Find the angle of intersection between the curves $r^n=a^n\cos n\theta$ and $r^n=b^n\sin n\theta$. — 6 L3 3

b) i) If $V=f(2x-3y,3y-4z,4z-2x)$ then find the value of $6\frac{\partial V}{\partial x}+4\frac{\partial V}{\partial y}+3\frac{\partial V}{\partial z}$.

ii) Find $\frac{dy}{dx}$ from the implicit function $x\sin(x-y)-(x+y)=0$. — 7 L2 3

c) Find the extreme value of the function $f(x,y)=x^3y^2(1-x-y)$. — 7 L3 4

6. a) Find the radius of curvature of the curve $x=6t^2-3t^4$, $y=8t^3$. — 6 L2 3

b) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$ — 7 L3 4

c) Expand the function $f(x,y)=\log(1+x-y)$ as a Maclaurin's series upto second degree terms. — 7 L2 4

Unit – III

7. a) Using Gamma function evaluate $\int_0^1 x^2\left(\log\left(\frac{1}{x}\right)\right)^3 dx$. — 6 L1 5

b) Prove that $\beta(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. — 7 L2 5

c) By changing the order of integration, evaluate $\int_0^3\int_1^{\sqrt{4-y}}(x+y)\,dx\,dy$. — 7 L3 5

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NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

First Semester B.E. (Credit System) Degree Examinations

April - May 2022

21MA101 - ENGINEERING MATHEMATICS - I

ation: 3 Hours — Max. Marks: 100

*te: Answer **Five full** questions choosing **Two full** questions from **Unit – I** & **Unit – II each** and **One full** question from **Unit – III**.*

	Unit – I	Marks	BT*	CO*	PO*
a)	Find the rank of the matrix $A=\begin{bmatrix}3&4&5&6&7\\4&5&6&7&8\\5&6&7&8&9\\10&11&12&13&14\end{bmatrix}$ using elementary row transformation.	6	L*2	1	1
b)	Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}3&1&4\\0&2&6\\0&0&5\end{bmatrix}$	7	L2	1	1
c)	Obtain the Maclaurin's series expansion for $f(x)=(1+x)^{\frac{1}{2}}$ up to the terms containing x^4.	7	L1	2	1
a)	Using Power method, find the dominant eigen value and corresponding eigen vector of the matrix $\begin{bmatrix}1&2&-1\\2&1&1\\1&-1&2\end{bmatrix}$ starting with an initial eigen vector as $\begin{bmatrix}1\\0\\0\end{bmatrix}$. Perform five iterations.	6	L1	1	1
b)	Using Gauss-Seidel method solve the given system of linear equations: $6x+y+z=105$, $4x+8y+3z=155$, $5x+4y-10z=65$. Take $\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$ as an initial approximation and carry out three iterations.	7	L3	1	2
c)	State D'Alembert's ratio test. Test the convergence of the series: $\sum\frac{1.3.5.7.........(2n-1)}{4.7.10........(3n+1)}$	7	L2	2	2

	c)	Obtain the Maclaurin's expansion of e^{sinx} upto third degree terms.	7	L3	2

Unit – III

5.	a)	With usual notation prove that $tan\varphi = r\frac{d\theta}{dr}$.	7	L3	3
	b)	Show that the radius of curvature at the point $(\frac{3a}{2}, \frac{3a}{2})$ of the folium $x^3 + y^3 = 3axy$ is $\frac{3a}{8\sqrt{2}}$	6	L2	3
	c)	State and prove Cauchy's mean value theorem.			
6.	a)	Find $\frac{ds}{d\theta}, \frac{ds}{dx}, \frac{ds}{dy}$ for the cycloid $x = a(\theta - sin\theta), y = a(1 - cos\theta)$	7	L3	3
	b)	i) With usual notation prove that $\rho = \frac{(1+y_1^2)^{3/2}}{y_2}$ ii) Find the radius of curvature for the curve $y = alog\frac{x}{a}$	7	L2	3
	c)	State and prove Lagrange's mean value theorem.	6	L1	3

Unit – IV

7.	a)	i) If $z = yf(x^2 - y^2)$ then show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{zx}{y}$. ii) If $z = x^2y - xsinxy$ find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.	6	L3	4
	b)	If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at (1,-1,0)	7	L3	4
	c)	Suppose a closed rectangular box has length twice its breadth and has constant volume V. Determine the dimension of the box requiring least surface area.	7	L2	4
8.	a)	i) If u is a homogeneous function of degree n prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$. ii) If $sinu = \frac{x^2+y^2}{x+y}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = tanu$.	6	L1	4
	b)	Find total derivative of $u = xy + yz + zx$, when $x = tcost$, $y = tsint, z = t$ at $\frac{\pi}{4}$.	7	L2	4
	c)	Expand $f(x,y) = \tan^{-1}(\frac{y}{x})$ in powers of $(x-1)$ and $(y-1)$ upto second-degree terms.	7	L3	4

Unit – V

9.	a)	Obtain the reduction formula for $\int \sin^n x\, dx$. Hence obtain $\int_0^{\pi/2} \sin^n x dx$.	7	L2	5
	b)	Trace the curve $y^2(a - x) = x^3, a > 0$.	7	L3	5
	c)	Find the volume of the solid generated by the revolution of the cardioid $r = a(1 + cos\theta)$ about the initial line.	6	L2	5
10.	a)	Evaluate i) $\int_0^{\pi} \frac{\sin^4\theta}{(1+cos\theta)^2} d\theta$ ii) $\int_0^{2a} x^2\sqrt{2ax - x^2}\, dx$	7	L2	5
	b)	Trace $r^2 = a^2cos2\theta$.	7	L3	5
	c)	Find the surface area generated by revolving asteroid $x = a\sin^3 t, y = a\cos^3 t$ about the initial line.	6	L2	5

BT* Bloom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

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First Semester B.E. (Credit System) Degree Examinations

Supplementary Examinations - September 2022

20MA101 - ENGINEERING MATHEMATICS - I

ition: 3 Hours — Max. Marks: 100

Note: Answer ***Five full*** *questions choosing* ***One full*** *question from* ***each Unit.***

		Marks	BT*	CO*	PO*
	Unit – I				
a)	Define Rank of the matrix. Find rank of the following matrix using elementary row transformation $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$	6	L*2	1	2
b)	Test for consistency and solve the following system of equations by Gauss-elimination method $x_1 + x_2 - x_3 = 0$, $2x_1 - x_2 + x_3 = 3$, $4x_1 + 2x_2 - 2x_3 = 2$	7	L3	1	2
c)	Find Eigen value and Eigen vector of the matrix $\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$	7	L3	1	2
a)	Determine the largest Eigen value and the corresponding Eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ using Rayleigh's power method, with initial approximation $[1 \quad 0 \quad 0]^T$, carry out four iterations.	7	L3	1	2
b)	Find matrix P which diagonalizes the matrix A=$\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$. Verify $P^{-1}AP = D$ where D is the diagonal matrix.	7	L2	1	2
c)	Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$ to canonical form.	6	L2	1	2
	Unit – II				
a)	Test for convergence of the series $\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \cdots$.	6	L2	2	2
b)	Examine convergence or divergence of the series $\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \cdots$, $x > 0$	7	L2	2	2
c)	Obtain Taylor's series expansion of $\log(\sec x)$ about the point $x = \frac{\pi}{3}$ upto third degree terms.	7	L3	2	2
a)	Test for convergence i) $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$ ii) $\sum \frac{\sqrt{n}}{n^2+1}$	6	L2	2	2
b)	State Cauchy's root test and test the convergence of $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \cdots, x > 0$	7	L2	2	2

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Unit – III

7. a)	Evaluate $\int_1^6\int_2^3\int_0^2 5x^2y^2z^3\ dxdydz$.	6	L2	5	1
b)	Prove that $\beta(m,n)=\dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.	7	L2	5	2
c)	Evaluate $\int_0^1\int_x^{\sqrt{x}} 5xy\ dydx$ by changing the order of integration.	7	L2	5	1
8. a)	Prove that $\int_0^{\frac{\pi}{2}}\sqrt{\sin\theta}\,d\theta \int_0^{\frac{\pi}{2}}\dfrac{1}{\sqrt{\sin\theta}}\,d\theta = \pi$.	6	L2	5	2
b)	Evaluate $\int_0^1 x^7(1-x^4)^3dx$ using Beta and Gamma functions.	7	L2	5	1
c)	Evaluate $\int_0^1\int_0^{\sqrt{1-y^2}} 2(x^2+y^2)\ dydx$ by changing to polar coordinates.	7	L2	5	1

BT* Bloom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

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	b) Using Rayleighs' power method obtain the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$,select $[1 \quad 0 \quad 0]^T$ as the initial eigen vector and carry out 5 iterations.	7	L1	1	1
	c) Write Maclaurin's series expansion of $f(x)=e^x \cos x$ up to third degree terms.	7	L2	2	2
	Unit – II				
4.	a) Given $u = 10 \sin\left(\frac{x}{y}\right)$, $x = 3e^{2t}$ and $y = 5t^2$, find $\frac{du}{dt}$ as a function of t.	6	L1	4	2
	b) If ρ is the radius of curvature at any point p on the parabola $y^2 = 4ax$ and s is its focus, then show that ρ^2 varies as $(sp)^3$.	7	L2	3	1
	c) With usual notation prove that $\tan\phi = r\frac{d\theta}{dr}$.	7	L2	3	2
5.	a) Expand the function $f(x,y)=(\cos x)(\cos y)$ up to second degree terms.	6	L1	4	1
	b) Examine the following function for extreme values $u = x^4 + y^4 - x^2 - y^2 + 1$.	7	L2	4	1
	c) State and prove Cauchy's mean value theorem.	7	L2	3	1
6.	a) The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$ .In an experiment carried out to find the value of g ,errors of 1.5% and 0.5% are possible in values of l and T respectively. Find the error in the calculated value of g .	6	L2	4	2
	b) If $x = r\cos\theta$, $y = r\sin\theta$,find $J = \frac{\partial(x,y)}{\partial(r,\theta)}$, $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$ and show that $JJ' = 1$.	7	L1	4	1
	c) With usual notations prove that $\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2}$.	7	L2	3	1

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First Semester B.E. (Credit System) Degree Examinations

September – October 2022

21MA101 - ENGINEERING MATHEMATICS - I

Duration: 3 Hours Max. Marks: 100

*Note: Answer **Five full** questions choosing **Two full** questions from **Unit – I & Unit – II each** and **One full** question from **Unit – III**.*

Unit – I

Q.		Question	Marks	BT*	CO*	PO*
1.	a)	Find the rank of the following matrix using elementary row transformations $\begin{bmatrix} 2 & 7 & 1 & -2 \\ 1 & 3 & 1 & 4 \\ -3 & 0 & -2 & 1 \\ 0 & -3 & 1 & 5 \end{bmatrix}$.	6	L*1	1	1
	b)	Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$	7	L2	1	1
	c)	(i) State Cauchy's root test. (ii) Test for convergence of the series $\sum \left(\frac{n+2}{n+3}\right)^n x^n, x > 0$.	7	L2	2	2
2.	a)	Solve the system of equations given below by Gauss - Seidel Method. $6x+15y+2z=72$ $27x+6y-z=85$, $x+y+54z=110$ take $x^{(0)}=0, y^{(0)}=0, z^{(0)}=0$, carry out three iterations.	6	L1	1	1
	b)	Test for convergence of the series $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\ldots\ldots\ldots \infty, x>0$.	7	L2	2	2
	c)	Test for consistency and solve the system of equations $x-4y+5z=8$ $3x+7y-z=2$ by Gauss elimination method. $x+15y-11z=-14$	7	L2	1	1
3.	a)	Test for the convergence of the series (i) $\sum \frac{n^3}{3^n}$, (ii) $\frac{1}{2.3}+\frac{1}{3.4}+\frac{1}{4.5}+\frac{1}{5.6}+\ldots\ldots\ldots \infty$	6	L2	2	1